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LETTER TO THE EDITOR

Disordered electron system with long-range correlated impurities*

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Abstract. We consider a d-dimensional electron gas in the presence of random potentials with Gaussian long-range correlations decaying with a power law $|x|^{-d-\sigma}$. We use the method of renormalised perturbation theory with dimensional regularisation and minimal subtraction to study the singular behaviour of a theory where the generating functional is the configurational average of the replicated vacuum amplitude. Calculations are carried out to one-loop order in a double expansion in $\varepsilon = 4 - d$ *and* σ. We obtain a new long-range fixed point for $\sigma < 0$, $2\sigma < \varepsilon < \sigma$ that is unstable along one direction in parameter space and it is interpreted as describing the localisation transition at high dimensionality. From the solution of the RG equation we derive the scaling law for the conductivity.

We study the singular behaviour of non-interacting electrons in d-dimensions, in the presence of random potentials $V(x)$ with zero mean and Gaussian, long-range correlations:

$$\langle V(x) V(x') \rangle = W_S \delta(x - x') + W_L |x - x'|^{-d-\sigma} \tag{1}$$

where the bracket indicates a configurational average.

A similar kind of quenched randomness has been previously studied in magnetic systems (Weinrib and Halperin 1983), but for disordered electrons all investigations until now have been confined to short-range correlations, $W_L = 0$. The most salient result obtained in this case is the scaling theory for the localisation transition by Abrahams *et al* (1979), while other investigations have focused on the formulation of a Lagrangian field theory. After Wegner (1979) and Schäffer and Wegner (1980), most authors derived field theories that describe the ordered, insulating phase by mapping the localisation problem onto a nonlinear σ-model with non-compact symmetry. This ordered phase would be characterised by a non-vanishing order parameter of a complicated nature (McKane and Stone 1981), just as in an ordinary magnet the compact nonlinear σ-model describes the low temperature phase with non-vanishing magnetisation (Amit 1984). In the work by Efetov *et al* (1980) Grassmann anticommuting fields were used to represent the electrons, although the use of boson or fermion fields is immaterial in the absence of electron–electron interactions because scattering with random impurities does not mix frequencies. When Coulomb interactions are taken into account, it is essential to use anticommuting fields in the derivation of the

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effective Lagrangian (Finkel'shtein 1983, 1984). Although the replica method was used all through this work to calculate the configurational averaged free energy, this can be avoided in the case of non-interacting electrons by using supersymmetry (or graded symmetry) methods to obtain the mapping to the nonlinear σ-model (Zirnbauer 1986). In all these papers, renormalisation group calculations were performed at dimensionality d close to the critical value for the nonlinear σ-model, $d_c = 2$.

Along different lines, one of us (Theumann 1983) pointed out that the generating functional for a system of disordered electrons is just the configurational average of the replicated grand partition function, or vacuum amplitude at $T=0$, $\langle Z^n \rangle$, expressed as a functional integral over Grassmann fields in the $n=0$ limit (Edwards 1975). The singular behaviour of this theory should describe the disordered, conducting phase, just as the φ^4-theory of the n-vector model describes a ferromagnet for $T \geqslant T^c$ (Amit 1984). Renormalisation of the theory at $d = 2+\varepsilon$ reproduced results analogous to the nonlinear σ-model.

In the present letter we follow a similar approach to study the properties of non-interacting electrons in the presence of impurities characterised by the long-range correlations of (1). By using the replica method we write the generating functional (Theumann 1983):

$$\langle Z^n \rangle \equiv Z_n = \int \prod_\alpha D\psi_\alpha^+ \psi_\alpha \, \mathrm{e}^{(A_0 + A_1)} \tag{2}$$

with

$$A_0 = \sum_\alpha \int \mathrm{d}\boldsymbol{k}\, \mathrm{d}\omega\, \Gamma^{(0)}_{\alpha\alpha}(\boldsymbol{k}, \omega) \psi_\alpha^+(\boldsymbol{k}, \omega) \psi_\alpha(\boldsymbol{k}, \omega) \tag{3}$$

$$A_1 = \frac{1}{4} \int \mathrm{d}\boldsymbol{k}\, \mathrm{d}\boldsymbol{k}'\, \mathrm{d}\boldsymbol{q}\, [W_S + W_L q^\sigma] \times \int \mathrm{d}\omega\, \mathrm{d}\omega' \sum_\alpha \psi_\alpha^+(\boldsymbol{k}+\boldsymbol{q}, \omega) \psi_\alpha(\boldsymbol{k}, \omega) \sum_{\alpha'} \psi_{\alpha'}^+(\boldsymbol{k}'-\boldsymbol{q}, \omega') \psi_{\alpha'}(\boldsymbol{k}', \omega') \tag{4}$$

and where $\psi_\alpha(\boldsymbol{k}, \omega)$ is a complex Grassman variable, while the replica indices α, α' run from 1 to n. All correlation functions can be expanded in a diagrammatic series in powers of W_S and W_L with the unperturbed inverse propagator:

$$\Gamma^{(0)}_{\alpha\alpha}(\boldsymbol{k}, \omega) = \tfrac{1}{2}k^2 - E_0 - \omega - \mathrm{i}\eta \operatorname{sgn}(\omega) \tag{5}$$

where E_0 is the bare Fermi energy, that in this problem plays the role of a negative 'mass'. The use of anticommuting variables is not essential in dealing with non-interacting electrons but it satisfies the symmetry requirements for a fermion system and it is the correct formulation for further extensions to interacting particles.

The perturbation expansion presents the usual divergencies that can be handled with the method of dimensional regularisation and renormalisation by minimal subtraction of dimensional poles (Amit 1984). A dimensional analysis of (4) tells us that W_S and W_L have different dimensions $[W_S] = \Lambda^{4-d}$ and $[W_L] = \Lambda^{4-d-\sigma}$, for Λ an inverse length, so we would not have a single critical dimension at which the degree of divergence of the correlation functions are independent of the order of calculation in perturbation theory. A way out of this situation is to renormalise in a double expansion in $\varepsilon = 4 - d$ *and* σ. It has been shown in a previous paper (Theumann 1989), from now on referred to as I, how to achieve this double expansion within the context of dimensional regularisation in the case of the m-vector model, and the same method of renormalisation by minimal subtraction will be followed here to first order in the loop expansion.

We write $W_S = U_S\kappa^{4-d}$ and $W_L = U_L\kappa^{4-d-\sigma}$ where κ is the scale parameter with dimension of an inverse length and the U's are dimensionless couplings. Singular contributions occur only for the inverse propagator and the four-point vertex, and they can be read directly from I. They are:

$$\Gamma_{\alpha\alpha}(\boldsymbol{k}, \omega) = \Gamma^{(0)}_{\alpha\alpha}(\boldsymbol{k}, \omega) + \kappa^2 \tfrac{1}{2}[U_S L_0 + U_L L_1] \tag{6}$$

$$\Gamma^4_{\alpha\beta}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{q}) = -\kappa^{\varepsilon}\{U_S + 2U_S^2 I_0 + 3U_S U_L I_1 + U_L^2 I_2\} - \kappa^{\varepsilon-\sigma} q^{\sigma}\{U_L + U_L U_S I_0 + U_L^2 I_1\}. \tag{7}$$

We do not need in the present problem to consider the two-point function with one insertion $\Gamma^{2,1}_{\alpha\alpha}$, because the system is not driven to criticality by a vanishing mass term (McKane and Stone 1981). In the localisation problem criticality is a function of the disorder (coupling constants), and it will be described by the infrared *unstable* fixed points of the renormalised couplings (Theumann 1983).

In (7) we indicate by I_μ (L_μ) dimensionless integrals with two (one) propagators and with μ long-range vertices, $\mu = 0$, 1 or 2. The surface S_d of the d-dimensional unit sphere is absorbed in the definition of the interactions. We obtain:

$$L_\mu(\boldsymbol{k}, \omega) = \frac{1}{S_d}\int \mathrm{d}\boldsymbol{q}\, q^{\mu\sigma}\left\{\frac{1}{2}\left|\boldsymbol{q} + \frac{\boldsymbol{k}}{\kappa}\right|^2 - \theta_\omega\right\}^{-1} \tag{8}$$

where we call:

$$\theta_\omega = \kappa^{-2}(E_0 + \omega + \mathrm{i}\eta \operatorname{sgn}\omega). \tag{9}$$

In order to evaluate the singular contributions to $\Gamma^4_{\alpha\beta}$ it is sufficient to consider the integrals at zero external momenta; then

$$I_\mu = \int_0^\infty \mathrm{d}y\, y[(d+\mu\sigma)/2 - 1]\frac{1}{(y-\theta_\omega)(y-\theta_{\omega'})}. \tag{10}$$

The method proposed in I consists in minimally subtracting the dimensional poles in ε *and* σ, while terms of the type (σ/ε) must be considered 'regular' and do not need to be subtracted. This means that one takes simultaneously the limits $\sigma \to 0$ and $\varepsilon \to 0$, keeping constant the ratio σ/ε.

The evaluation of $L_\mu(\boldsymbol{k}, \omega)$ is standard and it gives:

$$L_\mu(\boldsymbol{k}, \omega) = \Gamma\left(-1 + \frac{\varepsilon'}{2}\right)\Gamma\left(2 - \frac{\varepsilon'}{2}\right)(-\theta_\omega)^{1-\varepsilon'/2} + \left(\frac{k}{\kappa}\right)^2 \frac{\mu\sigma}{d}\Gamma\left(\frac{\varepsilon'}{2}\right)(-\theta_\omega)^{-\varepsilon'/2} + \mathrm{O}(k^4) \tag{11}$$

where $\varepsilon' = \varepsilon - \mu\sigma$ and

$$(-\theta_\omega)^\gamma = \left(\frac{E_0 + \omega}{\kappa^2}\right)^\gamma \mathrm{e}^{-\mathrm{i}\pi\gamma \operatorname{sgn}\omega}. \tag{12}$$

We notice first that, although we obtained a surprising dependence on the external momentum $\boldsymbol{k}$ at one-loop order, the dimensional pole is affected by an extra factor at σ and it gives a regular contribution that does not require a subtraction. We obtain for the singular contribution to the inverse propagator, from (6) and (11):

$$\Gamma_{\alpha\alpha}(\boldsymbol{k}, \omega) \sim \frac{k^2}{2} - (E_0 + \omega)\left(1 + U_0\frac{4}{\varepsilon} + U_1\frac{4}{\varepsilon - \sigma}\right) - 2\mathrm{i}\pi(E_0 + \omega)(U_0 + U_1)\operatorname{sgn}\omega. \tag{13}$$

In dealing with fermions it is essential to have a non-vanishing imaginary part in the propagator, no matter how small, in order to obtain the correct analytic properties. On the other hand we are free to renormalise at the value more convenient for ω; then we choose $\omega + E_0 = 0^+$ to obtain from (13):

$$\Gamma_{\alpha\alpha}(\boldsymbol{k}, \omega) = \frac{k^2}{2} - \mathrm{i}\frac{1}{\tau}\operatorname{sgn}\omega \tag{14}$$

with

$$\frac{1}{\tau} = 2\pi(E_0 + \omega)(U_0 + U_1) \to 0^+. \tag{15}$$

The inverse propagator is now free of singularities and the field renormalisation constant keeps the value $Z_\varphi = 1$.

We proceed now to the evaluation of the integrals in (10). The standard procedure now gives:

$$I_\mu = B\left(\frac{d}{2}, \frac{\varepsilon'}{2}\right)\int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2\,(1 - x_1 - x_2)^{-1-\mu\sigma/2}\{-x_1\theta_\omega - x_2\theta_{\omega'}\}^{-\varepsilon'/2}. \tag{16}$$

Then the singular part is independent of the value of $\theta_\omega = (\mathrm{i}/\tau)\operatorname{sgn}\omega$, so far as it is finite to prevent infrared singularities. We obtain:

$$[I_\mu]_{\mathrm{sing}} \sim \frac{2}{\varepsilon - \mu\sigma}. \tag{17}$$

From (14) and (17) we find that only the $\Gamma^{(4)}_{\alpha\beta}$ in (7) requires renormalisation; then we expand the coupling constants U_{S}, U_{L} in terms of dimensionless renormalised couplings λ_{S}, λ_{L}:

$$\begin{aligned} U_{\mathrm{S}} &= \lambda_{\mathrm{S}} - \frac{4}{\varepsilon}\lambda_{\mathrm{S}}^2 - \frac{6}{\varepsilon - \sigma}\lambda_{\mathrm{S}}\lambda_{\mathrm{L}} - \frac{2}{\varepsilon - 2\sigma}\lambda_{\mathrm{L}}^2 \\ U_{\mathrm{L}} &= \lambda_{\mathrm{L}} - \frac{2}{\varepsilon}\lambda_{\mathrm{L}}\lambda_{\mathrm{S}} - \frac{2}{\varepsilon - \sigma}\lambda_{\mathrm{L}}^2. \end{aligned} \tag{18}$$

Equation (18) gives for the β-functions, $\beta_i = [\kappa\,\partial\lambda_i/\partial\kappa]_{W_j}$, obtained by varying λ_i at constant-dimensional couplings $\{W_j\}$:

$$\begin{aligned} \beta_{\mathrm{S}} &= -\varepsilon\lambda_{\mathrm{S}} - 4\lambda_{\mathrm{S}}^2 - 6\lambda_{\mathrm{S}}\lambda_{\mathrm{L}} - 2\lambda_{\mathrm{L}}^2 \\ \beta_{\mathrm{L}} &= -(\varepsilon - \sigma)\lambda_{\mathrm{L}} - 2\lambda_{\mathrm{S}}\lambda_{\mathrm{L}} - 2\lambda_{\mathrm{L}}^2. \end{aligned} \tag{19}$$

The set of differential equations in (19) describe the flow of the couplings under a change of scale and they will be approximately solved by linearisation around the fixed points (FP) that satisfy $\beta_i(\lambda_j^* = 0)$. Recalling that, from (1), the physical couplings can only take positive values, we obtain besides the trivial fixed point $\lambda_{0\mathrm{L}}^* = \lambda_{0\mathrm{S}}^* = 0$:

1. short-range FP

$$\lambda_{1\mathrm{L}}^* = 0 \qquad \lambda_{1\mathrm{S}}^* = -\varepsilon/4 \tag{20}$$

physical and unstable for $d > 4$;

2. long-range FP

$$\lambda_{2L}^* = \frac{(\varepsilon - \sigma)(\varepsilon - 2\sigma)}{2\sigma}$$
$$\lambda_{2S}^* = -\frac{(\varepsilon - \sigma)^2}{2\sigma} \tag{21}$$

physical and unstable for $\sigma < 0, 4 + |\sigma| < d < 4 + 2|\sigma|$.

These results tell us that for negative (positive) values of σ the system will have long-range (short-range) behaviour, as it would be predicted from (1). The stability was calculated from the solution of the linearised flow equations:

$$\kappa \frac{d\Delta_0}{d\kappa} = -(\varepsilon + 8\lambda_0^*)\Delta_0 + \sigma\Delta_L$$
$$\kappa \frac{d\Delta_L}{d\kappa} = -(\varepsilon - \sigma + 2\lambda_0^*)\Delta_L - 2\lambda_L^*\Delta_0 \tag{22}$$

where $\Delta_L = \lambda_L - \lambda_L^*$ and $\Delta_0 = \lambda_L + \lambda_S - \lambda_L^* - \lambda_S^*$. The solution of (22) close to the short-range FP in (20) gives:

$$\Delta_0^1(\kappa) \sim C_0 \kappa^{\varepsilon}$$
$$\Delta_L^1(\kappa) \sim C_L \kappa^{(2\sigma - \varepsilon)/2} \tag{23}$$

that is infrared unstable along U_0 for $\varepsilon < 0$.

In the vicinity of the long-range FP we diagonalise (22) by introducing the potentials:

$$V_\pm = \alpha_\pm(\lambda_L + \lambda_S) + \sigma\lambda_L \sim V_\pm^* + C\kappa^{\alpha_\pm} \tag{24}$$

where the $\alpha_\pm$ are the eigenvalues:

$$\alpha_\pm = \tfrac{3}{2}\varepsilon - 2\sigma \pm \tfrac{1}{2}[(3\varepsilon - 4\sigma)^2 + 4(\sigma - \varepsilon)(\varepsilon - 2\sigma)]^{1/2} \tag{25}$$

with $\alpha_+ > 0$ and $\alpha_- < 0$. Then the long-range FP is also infrared unstable along one direction in the physical region $2\sigma < \varepsilon < \sigma, \sigma < 0$.

To give an interpretation to the previous results we first notice that precisely these two fixed points have been discarded by Weinrib and Halperin (1983) in their description of the random ferromagnet. This is because in the magnetic transition criticality is a function of a relevant, thus unstable, temperature term, while the other potentials should flow to an attractive fixed-point value.

On the other hand, we are studying here a non-interacting random electron system whose singular behaviour can only originate in the localisation transition. Now, in this transition the system is not driven to criticality by a vanishing temperature term but rather by a critical value of the disorder as measured by the variance in (1). It is then the potentials λ_S and λ_L that should have one direction of instability in parameter space. From (23) and (24) we obtain the following picture: (i) for $\sigma > 0$ and $\varepsilon < 0$ the singular behaviour would be of the short-range type and described by the approximate flow trajectories in (23); (ii) for $\sigma < 0$ and $\sigma < \varepsilon < 0$, the short-range FP is physical but unstable along two directions, while the long-range FP is unphysical; (iii) for $\sigma < 0$ and $2\sigma < \varepsilon < \sigma$ there would be singular behaviour as described by the physical long-range FP with one direction of instability from (24) and (25), while the short-range FP remains unstable along two directions.

The problem with this interpretation is that it would predict a localisation transition when $d > 4$ for short-range correlations with $\lambda_L = 0$, which contradicts all known results in the field until now. To solve this puzzle we look at the one-loop correction to the DC conductivity:

$$\Delta\sigma_{DC} = \kappa^{d-2} \int dk\, dk'\, \boldsymbol{k}\cdot\boldsymbol{k}'[U_S + U_L|\boldsymbol{k}-\boldsymbol{k}'|^{\sigma}]G(k, 0^+)G(k, 0^-)G(k', 0^+)G(k', 0^-) \tag{26}$$

where $G(k, \omega) = \kappa^2\Gamma^{-1}_{\alpha\alpha}(k, \omega)$ and, from (14), it depends only on $|\boldsymbol{k}|$. The angular integral in (26) gives a vanishing short-range contribution while the long-range integral is non-zero and proportional to σ.

As our renormalisation group calculation is performed to one-loop order, we argue consistently that the short-range FP in (20) that would give $\Delta\sigma^*_{DC}(\mathrm{SR}) = 0$ cannot be interpreted as describing localisation, in agreement with known results. On the other hand, the relevant long-range FP in (21) that gives $\Delta\sigma^*_{DC}(\mathrm{LR}) \neq 0$ would describe a localisation transition for $4 + |\sigma| < d < 4 + 2|\sigma|$.

In the vicinity of this fixed point we can write the renormalisation group equation for σ_{DC}:

$$\left(\kappa\frac{\partial}{\partial\kappa} + \beta_+\frac{\partial}{\partial v_+} + \beta_-\frac{\partial}{\partial v_-}\right)\sigma_{DC} = 0 \tag{27}$$

where, from (24) and calling $\Delta_\pm = v_\pm - v^*_\pm$:

$$\beta_\pm = \kappa\frac{d}{d\kappa}v_\pm \sim \alpha_\pm\Delta_\pm. \tag{28}$$

Solving (27) by the method of characteristics gives:

$$\sigma_{DC} = \Phi[\kappa^{-1}\Delta^{1/\alpha_+};\ \kappa^{-1}\Delta^{1/\alpha_-}] \tag{29}$$

where $\Phi(x, y)$ is an arbitrary function of two variables. From (29) we can identify the correlation length:

$$\xi = \kappa^{-1}\Delta_-^{1/\alpha_-} = \kappa^{-1}(v_- - v^*_-)^{-\nu} \tag{30}$$

with the correlation length exponent $\nu^{-1} = |\alpha_-|$, and from (29) we can write the scaling relation:

$$\sigma_{DC} \sim \xi^{2-d}\Phi[\Delta_+^{1/\alpha_+}\Delta_-^{\nu};\ 1]. \tag{31}$$

From (31) we obtain that close to the critical value v_c of v_- in (24) the DC conductivity will vanish as:

$$\sigma_{DC} \sim |v_- - v_c|^{(d-2)\nu} \tag{32}$$

for $4 + |\sigma| < d < 4 + 2|\sigma|$, $\sigma < 0$.

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